Computation of the symmetries of a rational ruled surface

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The *symmetries* of a surface S are the orthogonal transformations $f(x) = \mathbf{Q}x + \mathbf{b}$, $x \in R^3$, $\mathbf{Q} \in R^{3\times 3}$, $\mathbf{b} \in R^3$, leaving S invariant. Examples of symmetries are the symmetries with respect to a plane (planar symmetries), symmetries with respect to a line (axial symmetries), symmetries with respect to a point (central symmetry) or rotational symmetries (rotations leaving a surface invariant). Given a ruled rational surface, defined by means of a proper (i.e. generically injective) parametrization

$$\mathbf{x}(t, s) = \mathbf{p}(t) + s\mathbf{q}(t),$$

we provide an algorithm to determine the *symmetries* of the surface. The key idea is to prove that any symmetry of the surface corresponds to a birational transformation $\varphi(t, s) = (\varphi_1(t, s), \varphi_2(t, s))$ in the parameter space (i.e. the t, s-plane) whose structure can be predicted, and which can be explicitly computed. The method, up to a certain extent, generalizes to ruled surfaces some ideas used in [1, 2] to compute the symmetries of rational curves and polynomially parametrized surfaces.

Joint work with: Emily Quintero.

References

[1] J.G. Alcázar, C. Hermoso. Involutions of Polynomially Parametrized Surfaces. *Journal of Computational and Applied Mathematics*, 294: 23–38, 2016.

[2] J.G. Alcázar, C. Hermoso, G. Muntingh. Symmetry detection of rational space curves from their curvature and torsion. *Computer Aided Geometric Design*, pages 51–65, 1781.