Interpolatory pointwise estimates for monotone polynomial approximation

I. A. Shevchuk
Taras Shevchenko National University of Kyiv
shevchuk@univ.kiev.ua

Among the results we are going to discuss is the following Theorem 1, proved by DeVore and Yu [1] for $r \leq 2$, and by Kopotun, Leviatan and the author [2] for $r > 2$.

Let $\|\cdot\| := \|\cdot\|_{L_\infty[-1,1]}$. In particular, $\|g\| = \|g\|_{C[-1,1]}$, if $g \in C[-1,1]$. Denote by W^r, $r \in \mathbf{N}$, the (Sobolev) space of functions $f \in C[-1,1]$, having the $r-1$-st absolutely continuous derivative on $[-1,1]$, and such that $\|f^{(r)}\| < \infty$.

Theorem 1. *For every $r \in \mathbf{N}$ there is a constant $c(r)$, such that for each* **monotone** *function $f \in W^r$ there are a number $N = N(f,r)$ and a sequence $\{P_n\}_{n=N}^{\infty}$ of* **monotone** *algebraic polynomials of degree $\leq n$ on $[-1,1]$, satisfying*

$$\left\| \frac{f - P_n}{\varphi^r} \right\| \leq c(r) \frac{\|f^{(r)}\|}{n^r},$$

where $\varphi(x) := \sqrt{1-x^2}$.

Remark, $N = 1$ for $r \leq 2$, whereas N cannot be independent on f for $r > 2$.

References

[1] R. A. DeVore and X. M. Yu. Pointwise estimates for monotone polynomial approximation. *Constr. Approx.*, 1:323–331, 1985.

[2] K. A. Kopotun, D. Leviatan and I. A. Shevchuk. Interpolatory pointwise estimates for monotone polynomial approximation. *Journal of Mathematical Analysis and Applications*, 459:1260–1295, 2018.