Are the degrees of unconstrained and constrained approximation the same? (comonotonicity as example)

Dany Leviatan
Tel Aviv University
leviatan@tauex.tau.ac.il

Let $E_n(f)$ denote the degree of approximation of $f \in C[-1,1]$, by algebraic polynomials of degree $< n$, and assume that we know that for some $\alpha > 0$ and $N \geq 1$,

$$n^\alpha E_n(f) \leq 1, \quad n \geq N.$$

Suppose that f changes its monotonicity $s \geq 0$ times in $[-1,1]$. We are interested in what may be said about its degree of approximation by polynomials of degree $< n$ that are comonotone with f. In particular, if f changes its monotonicity at $Y_s := \{y_1, \dots, y_s\}$ ($Y_0 = \emptyset$) and the degree of comonotone approximation is denoted by $E_n(f, Y_s)$, we investigate when can one say that

$$n^\alpha E_n(f, Y_s) \leq c(\alpha, s, N), \quad n \geq N^*,$$

for some N^*. Clearly, N^*, if it exists at all (we prove it always does), depends on α, s and N. However, it turns out that for certain values of α, s and N, N^* depends also on Y_s and in some cases even on f itself.

Joint work with: I. A. Shevchuk.

References

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