Multigrid and multivariate subdivision

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We show that a variety of new interpolating and approximating iterative methods (subdivision schemes) developed to design curves and surfaces can be successfully used as grid transfer operators for multigrid methods.

Multigrid methods are efficient solvers for large ill-conditioned linear systems of equations with symmetric, positive definite system matrices. Any multigrid method consists of two main steps: a so-called smoother and a coarse grid correction, iterated until the remaining smaller linear system of equations is solved exactly. The smoother is a simple iterative solver such as Gauss-Seidel or weighted Jacobi, whose convergence is slow due to the ill-conditioning of the system matrix. The coarse grid correction step speeds up the convergence of the method and is a standard error reduction procedure performed at a coarser grid. The projection of the problem onto a coarser grid and the lifting of the error correction term to the finer grid is done, in our case, via the grid transfer operators based on multivariate subdivision schemes.

The analysis of the convergence and optimality of the corresponding subdivision based multigrid methods is done by relating the symbol analysis of the coarse grid correction for Toeplitz system matrices with the approximation properties of multivariate subdivision schemes. We show that the polynomial generation property and the stability of a subdivision scheme are crucial for convergence and optimality of the corresponding multigrid method. A stationary iterative method is called optimal whenever its convergence rate is linear and the computational cost of each iteration is proportional to the cost of a matrix vector product.

We illustrate our theoretical results with several examples of linear systems derived via discretizations of isotropic and anisotropic elliptic PDE's.

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