Optimal recovery of three times differentiable functions on a convex polytope inscribed in a sphere

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We consider the problem of global recovery on the class $W^3(P)$ of three times differentiable functions which have uniformly bounded third order derivatives in any direction on a d-dimensional convex polytope P inscribed in a sphere and containing its circumcenter. The information $I(f)$ known about each function $f \in W^3(P)$ is given by its values and gradients at the vertices of P. The recovery error is measured in the uniform norm on P. We prove the optimality on the class $W^3(P)$ of a certain quasi-interpolating recovery method among all non-adaptive global recovery methods which use the information $I(f)$.

This method was constructed earlier for the case of a d-dimensional simplex T in the work by the author and T.S. Sorokina in [1], where its optimality was proved for an analogous class $W^2(T)$ of twice differentiable functions. This recovery method is a quadratic polynomial over the simplex T defined by values and gradients of f at the vertices of T. As we showed in [1], it interpolates only values of f at the vertices of T. One also obtains a continuous spline when using this method on each simplex of a triangulation of a general polyhedral domain. Furthermore, it reproduces exactly any quadratic polynomial.

References

[1] S.V. Borodachov, T.S. Sorokina, An optimal multivariate spline method for recovery of twice differentiable multivariate functions, *BIT Numerical Mathematics 51(3), pages 497–511, 2011.*