Bias reduction in variational regularization

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Variational methods suffer from inevitable bias. The simplest example is ℓ^1-regularization, which leads to sparse solutions, but however affects the quantitative peak values. We present a two-step method to reduce bias in variational methods.

After solving the standard variational problem, the key idea is to add a consecutive debiasing step minimizing the data fidelity on an appropriate set, the so-called model manifold. Here, these spaces are defined by Bregman distances or infimal convolutions thereof, using the subgradient appearing in the optimality condition of the variational method. In particular, they lead to a decomposition of the overall bias into two parts, model and method bias.We propose to reduce the method bias thanks to the adapted model manifolds. The remaining model bias is considered as the unavoidable part of the bias, linked to the choice of regularization and hence the solution space of the variational method. The most popular example is the staircasing effect that occurs for total variation regularization due to the assumption of a piecewise constant solution.

For particular settings, such as anisotropic ℓ^1 and TV-type regularization, previously used debiasing techniques are shown to be special cases. The proposed approach is however easily applicable to a wider range of regularizations. The two-step debiasing is shown to be well-defined and to optimally reduce bias in a certain setting. We provide numerous examples and experiments to illustrate both the performance and the statistical behavior of the method. In addition to visual and PSNR-based evaluations, different notions of bias and variance decompositions are investigated in numerical studies.

Joint work with: Eva-Maria Brinkmann, Martin Burger & Julian Rasch.

References

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