Approximation of Functions Over Manifolds From Scattered Data by Moving Least-Squares

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We approximate a function defined over a d-dimensional manifold $\mathcal{M} \subset \mathbb{R}^n$ utilizing only noisy function values at noisy locations on the manifold. To produce the approximation we do not require any knowledge regarding the manifold other than its dimension d. The approximation scheme is based upon the Manifold Moving Least-Squares (MMLS) presented in [1] and is resistant to noise in both the domain $\mathcal{M}$ and function values. Furthermore, the approximant is shown to be smooth and of approximation order of $O(h^{m+1})$ for non-noisy data, where h is the mesh size with respect to $\mathcal{M}$, and m is the degree of the local polynomial approximation. In addition, the proposed algorithm is linear in time with respect to the ambient-space's dimension n, making it useful for cases where $d << n$. This assumption, that the high dimensional data is situated on (or near) a significantly lower dimensional manifold, is prevalent in many high dimensional problems. We put our algorithm to numerical tests against state-of-the-art algorithms for regression over manifolds and show its potential.

Joint work with: Yariv Aizenbud, David Levin.

References

[1] B. Sober and D. Levin. Manifolds' Projective Approximation Using The Moving Least-Squares (MMLS). arXiv preprint arXiv:1606.07104