Approximation of convex bodies by convex algebraic level surfaces

András Kroó
Alfréd Rényi Institute of Mathematics,
Hungarian Academy of Sciences
kroo@renyi.hu

We consider the problem of approximation of convex bodies in finite dimensional Euclidian spaces $\mathbb{R}^d$ by level surfaces of convex algebraic polynomials. This problem goes back to Minkowski who verified that the boundary of any convex body in $\mathbb{R}^d$ can be approximated arbitrarily well by a level surface of a convex analytic function. Subsequently, Hammer generalized this result showing that the approximation can be accomplished by using convex algebraic level surfaces. Recently, a quantitative version of Hammer's approximation theorem was given by the author showing that the order of approximation of convex bodies by convex algebraic level surfaces of degree $n$ is $O\left(\frac{1}{n}\right)$. Moreover, it was also verified that whenever the convex body is not smooth (that is there exists a point on its boundary with two distinct supporting hyper planes) then $\frac{1}{n}$ is essentially the sharp rate of approximation in this problem. Naturally, this raises the question wether this rate of approximation can be improved further when the convex body is smooth, i.e., it possesses a unique supporting hyper plane at each boundary point. It turns out that for any smooth convex body a $o\left(\frac{1}{n}\right)$ rate of convergence holds. In addition, if the body satisfies the condition of $C^2$-smoothness the rate of approximation is $O\left(\frac{1}{n^2}\right)$.

We will also review the similar problem of approximation by homogenous polynomial level surfaces.