Approximation of convex bodies by convex algebraic level surfaces

András Kroó
Alfréd Rényi Institute of Mathematics,
Hungarian Academy of Sciences
kroo@renyi.hu

We consider the problem of approximation of convex bodies in finite dimensional Euclidian spaces $\mathbf{R}^d$ by level surfaces of **convex** algebraic polynomials. This problem goes back to Minkowski who verified that the boundary of any convex body in $\mathbf{R}^d$ can be approximated arbitrarily well by a level surface of a *convex analytic* function. Subsequently, Hammer generalized this result showing that the approximation can be accomplished by using *convex algebraic* level surfaces. Recently, a quantitative version of Hammer's approximation theorem was given by the author showing that the order of approximation of convex bodies by convex algebraic level surfaces of degree n is $O(\frac{1}{n})$. Moreover, it was also verified that whenever the convex body is **not smooth** (that is there exists a point on its boundary with two distinct supporting hyper planes) then $\frac{1}{n}$ is essentially the sharp rate of approximation in this problem. Naturally, this raises the question wether this rate of approximation can be improved further when the convex body is **smooth**, i.e., it possesses a unique supporting hyper plane at each boundary point. It turns out that for any **smooth convex body** a $o(\frac{1}{n})$ rate of convergence holds. In addition, if the body satisfies the condition of C^2**-smoothness** the rate of approximation is $O(\frac{1}{n^2})$.

We will also review the similar problem of approximation by homogenous polynomial level surfaces.