Exact and efficient computations for Galerkin Boundary Element Methods

Norbert Warncke
Adwen Offshore
norbert.warncke@adwenoffshore.com

Boundary Element Methods (BEM) have recently had a renewed interest in the field of wind energy as they allow to model more of the unsteady flow phenomena around wind turbine airfoils than Blade Element Momentum theory. Though being computationally more complex, their costs are still significantly lower than CFD methods, placing them in a sweet-spot for the validation of turbine designs under various conditions (yaw, turbulent wind).

All analytic BEM methods used so far are based on the so-called Collocation Point Method [1], evaluating the influence of a surface panel on a target point only. This method can lead to numerical errors in even surprisingly simple situations (for instance, orientation-dependent results for the potential flow around the sphere). Errors for the meshes of a full wind turbine are even more difficult to obtain without a correct reference solution. By changing the BEM flow solver to a Galerkin method, it is expected that numerical errors decrease and, in particular, that the results are less sensitive to the mesh quality.

Galerkin BEMs have already successfully been implemented based on numerical quadrature [2, 3], albeit again at relatively high computational costs (wind turbine geometries require the recalculation of the system matrices every time step). To reduce these costs, Salles&Lenoir have recently published a fully analytic Galerkin method [4]. Their main idea is a decomposition of the arising 4D influence integrals into analytically solved line integrals, making use of the fact that the Green's function is homogeneous.

Based on the results of Lenoir and Salles, the aim of this work is to propose less computationally complex formulas for the analytic computation of the integral terms, and to have an efficient implementation for fully parallelised computations [5]. This approach then couples the advantages of the Galerkin methods (stability and theoretical proof of convergence) with efficiency. Numerical results of simple test cases will also be shown to support this approach.

Joint work with: Ioana CIOTIR (LMI - INSA Normandie), Antoine TONNOIR (LMI - INSA Normandie)

References

[1] Hess J. L. & Smith A. M. O. Calculation of potential flow about arbitrary bodies. *Progress in Aerospace Sciences*, 8, 1-138, 1967.

[2] Sauter S. A. & Schwab C. Quadrature for hp-Galerkin BEM in $\mathbb{R}$. *Numerische Mathematik*, 78(2), 211-258, 1997.

[3] Sauter S. A. & Schwab C. Boundary Element Methods. In: *Springer Series in Computational Mathematics*, vol 39, Springer, Berlin, Heidelberg

[4] Lenoir M. & Salles N. Evaluation of 3-d singular and nearly singular integrals in Galerkin BEM for thin layers. *SIAM Journal on Scientific Computing*, 34(6), A3057-A3078, 2012.

[5] Buchau A. & Tsafak S. M. & Hafla W. & Rucker W. M. Parallelization of a Fast Multipole Boundary Element Method with Cluster OpenMP. *IEEE Transactions on Magnetics*, 44(6), 1338-1341, 2008